YEAR 12 MATHEMATICS
METHODS UNIT 3

TEST 2
TERM 1, 2017
Test date: Friday 24th March

APPLECROSS
SENIOR HIGH SCHOOL

STUDENT NAME: Solutions

All working must be shown in the space provided. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than 1 mark, valid working or justification is required to receive full marks.

	Total	Result	
Section 1	20		%
Section 2	~~54~~ 48		
Total	~~74~~ 68		

Section 1: Resource – Free

Working time: ~~15~~ 20 minutes

Question 1 [2 marks]

Find y if $\frac{dy}{dx} = 3x^2 + x + 3$ and $y = 0$ when $x = 1$.

$y = x^3 + \frac{x^2}{2} + 3x + c$ ✓

$0 = 1 + \frac{1}{2} + 3 + c$

$c = -4\frac{1}{2}$

$y = x^3 + \frac{x^2}{2} + 3x - \frac{9}{2}$ ✓

Question 2 [2 marks]

Determine $\int \sqrt[3]{4x+3}\ dx$

$= \int (4x+3)^{1/3}\,dx$

$= \frac{3(4x+3)^{\frac{4}{3}}}{4 \times 4} + c$ ✓✓

$= \frac{3(4x+3)^{4/3}}{16} + c$

Question 3 [2 marks]

Find the antiderivative of $\frac{1}{(2x-1)^3}$ with respect to x.

$\int (2x-1)^{-3}dx = \frac{(2x-1)^{-2}}{-2 \times 2} + c$ ✓✓

$= -\frac{1}{4(2x-1)^2} + c$

Question 4 [2, 2, 2 = 6 marks]

Find

a) $\int \frac{5}{2}(\sqrt{x} - 1)^2\ dx$

$= \frac{5}{2}\int (x - 2\sqrt{x} + 1)\,dx$ ✓

$= \frac{5}{2}\left(\frac{x^2}{2} - \frac{4x^{3/2}}{3} + x\right) + c$ ✓

$= \frac{5x^2}{4} - \frac{20x^{3/2}}{6} + \frac{5x}{2} + c$

b) $\int 5x^2\,(3 + 2x^3)^4 dx$

$= \int \frac{5 \times 6x^2(3+2x^3)^4}{6}\,dx$ ✓

$= \frac{5}{6}(3+2x^3)^5 + C$ ✓

Question 4 [2, 2, 2 = 6 marks]

Find

a) $\int \frac{5}{2}(\sqrt{x}-1)^2\,dx$ $= \int \frac{5}{2}(x - 2\sqrt{x} + 1)\,dx$ ✓

$= \frac{5}{2}\left(\frac{x^2}{2} - \frac{2x^{3/2}}{3/2} + x\right) + c$

$= \frac{5x^2}{4} - \frac{20x^{3/2}}{6} + \frac{5x}{2} + c$ ✓

b) $\int 5x^2(3+2x^3)^4\,dx$

$= \frac{5}{6}\int 6x^2(3+2x^3)^4\,dx$ ✓

$= \frac{5}{6}\left(\frac{(3+2x^3)^5}{5}\right) + c$

$= \frac{5(3+2x^3)^5}{5\times 6}$ ✓

$= \frac{(3+2x^3)^5}{6} + c$

c) $\int \frac{x-x^3}{(3-2x^2+x^4)^5}\,dx$ $= \int (x-x^3)(3-2x^2+x^4)^{-5}\,dx$ ✓

$= -\frac{1}{4}\int(-4x+4x^3)(3-2x^2+x^4)^{-5}\,dx$

$= -\frac{1}{4}\frac{(3-2x^2+x^4)^{-4}}{-4} + c$

$= \frac{1}{16(3-2x^2+x^4)^4} + c$

Question 5 [3 marks] ✓

Find the derivative of $F(x)$ given that $F(x) = \int_1^{x^2+3}(2t-1)dt$

$\frac{d}{dx}\left(\int_1^{x^2+3}(2t-1)\,dt\right)$ ✓

$= (2(x^2+3) - 1)\times 2x$ ✓

$= 4x(x^2+3) - 2x$

$= 4x^3 + 10x$ ✓

F T of C

$F(x) = \int_a^{g(x)} f(x)\,dx$

$F'(x) = f(g(x)) \times g'(x)$

or you could find F(x) 1st by integrating and substituting then differentiate. Still gets there but takes a lot longer.

Question 6 [2, 3, 3 = 8 marks]

Determine these definite integrals:

a) $\int_{-1}^{1}(x+1)(x-2)dx$ $= \int_{-1}^{1}(x^2 - x - 2)\,dx$ ✓

$= \left[\frac{x^3}{3} - \frac{x^2}{2} - 2x\right]_{-1}^{1}$

$= \left(\frac{1}{3} - \frac{1}{2} - 2\right) - \left(-\frac{1}{3} - \frac{1}{2} + 2\right)$

$= -\frac{13}{6} - \frac{7}{6}$

$= -\frac{20}{6} = -\frac{10}{3}$ ✓

b) $\int_1^2\left(\frac{x+1}{x^3}\right)dx$

$= \int_1^2\left(\frac{x}{x^3} + \frac{1}{x^3}\right)dx$

$= \int_1^2 (x^{-2} + x^{-3})\,dx$ ✓

$= \left[\frac{x^{-1}}{-1} + \frac{x^{-2}}{-2}\right]_1^2$ ✓

$= \left[-\frac{1}{x} - \frac{1}{2x^2}\right]_1^2$

$= \left(-\frac{1}{2} - \frac{1}{2(4)}\right) - \left(-1 - \frac{1}{2}\right)$

$= -\frac{5}{8} - -\frac{12}{8}$

$= \frac{7}{8}$ ✓

c) $\int_{-1}^{0}\sqrt{1+x}\,dx$

$= \int_{-1}^{0}(1+x)^{1/2}\,dx$ ✓

$= \left[\frac{(1+x)^{3/2}}{3/2}\right]_{-1}^{0}$ ✓

$= \left[\frac{2(1+x)^{3/2}}{3}\right]_{-1}^{0}$

$= \frac{2}{3} - 0 = \frac{2}{3}$ ✓

END OF SECTION 1

YEAR 12 MATHEMATICS METHODS UNIT 3

TEST 2
TERM 1, 2017
Test date: Thursday 23rd March

APPLECROSS
SENIOR HIGH SCHOOL

STUDENT NAME: ______________________________

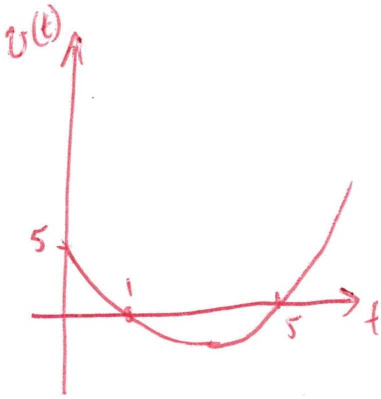
~~48~~ 45

All working must be shown in the space provided. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than 1 mark, valid working or justification is required to receive full marks.

Section 2: Resource – Rich
Working time: 38 minutes

To be provided by the student:
ClassPad and/or Scientific Calculators
1 sheet of A4–sized paper of notes, double-sided

Question 1 [2, 2, 2, 2 = 8 marks]



The velocity $v(t)$ in m/s of a particle travelling in a straight line is given by
$$v(t) = t^2 - 6t + 5$$

a) Determine the distance travelled in the 4th second. $t = 3$ to $t = 4$

$\text{dist} = \int_3^4 |v(t)|\,dt$ ✓

$= 3\frac{2}{3}\ m$ ✓

b) Determine the distance travelled in the interval $1 \le t \le 5$.

$\text{dist} = \int_1^5 |v(t)|\,dt$ ✓

$= 10\frac{2}{3}\ m$ ✓

c) If initially the particle has a displacement of $-10m$, what is the displacement when $t = 3$.

$v = t^2 - 6t + 5$

$x = \frac{t^3}{3} - 3t^2 + 5t + c$

at $t=0, x=-10$ $\therefore c = -10$ ✓

at $t=3$ $x = -13m$ ✓

OR

$x = -10 + \int_0^3 v(t)\,dt$ ✓

$= -10 + -3$ ✓

$= -13m$

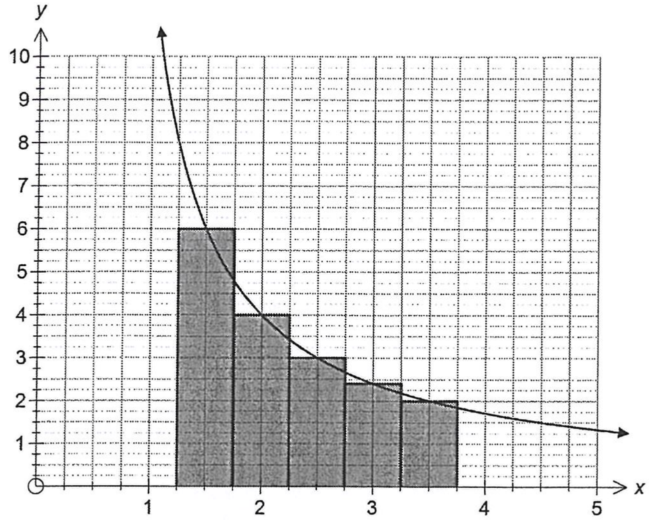
d) Calculate the acceleration when $= 2$.

$acc = v'(t) = 2t - 6$ ✓

at $t = 2$, $acc = -2 m/s^2$ ✓

Question 2 [3 marks]

The graph below shows the curve $y = f(x)$, where $f(3) = 2.4$.



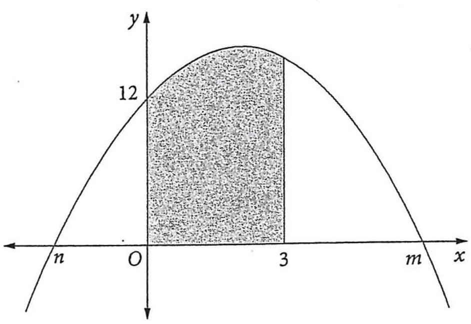
Use three of the five centred rectangles shown to estimate the shaded area under the curve from $x = 1.75$ to $x = 3.25$.

Area $= 0.5 \times 4 + 0.5 \times 3 + 0.5 \times 2.4$ or $0.5(4 + 3 + 2.4)$

$= 4.7 \text{ units}^2$ $= 4.7 \text{ units}^2$

Question 3 [5 marks]

Part of the graph of $f(x) = -x^2 + ax + 12$ is shown below:



If the area of the shaded section is 45 square units, determine the values of a, m and n, where m and n are the x intercepts of the graph of $y = f(x)$.

$\int_0^3 (-x^2 + ax + 12)\,dx = 45$

from class pad

$\frac{9a}{2} + 27 = 45$

$\therefore a = 4$

$\therefore f(x) = -x^2 + 4x + 12$

Solve $0 = -x^2 + 4x + 12$

$x = 6$ or -2

$\therefore n = -2$ and $m = 6$

Question 4 [2, 3= 5 marks]

The instantaneous rate with which the concentration C, in mg/KL, of a chemical compound in a river system changes with respect to time, t weeks, is modelled by the equation

$$\frac{dC}{dt} = \frac{1}{(t+0.5)^2} - 2t, \qquad for\ t \geq 0$$

The initial concentration was 9.3 mg/KL.

a) Determine the net change in concentration in the first week. $t=0$ to $t=1$

$\Delta C = \int_0^1 \frac{dC}{dt}\, dt$ ✓ OR $\Delta C = \left[-\frac{1}{t+0.5} - t^2\right]_0^1$ ✓

from classpad $= \frac{1}{3}$ mg/KL ✓ $= \frac{1}{3}$ mg/KL ✓

b) Find the maximum concentration and when this occurred.

max/min when $\frac{dC}{dt} = 0$

ie $\frac{1}{(t+0.5)^2} - 2t = 0$ ✓

at $t = 0.5$ wks

$C = \frac{-1}{t+0.5} - t^2 + c$

$t=0$, $C = 9.3 = \frac{-1}{0.5} + c$

$c = 9.3 + 2$ ✓

$c = 11.3$

$\therefore C = \frac{-1}{t+0.5} - t^2 + 11.3$

at $t = 0.5$, $C = 10.05$ mg/KL ✓

Question 5 [2, 2 = 4 marks]

Given that $f(x)$ is continuous everywhere and that $\int_{-4}^{6} f(x)\, dx = 10$, find

a) $\int_{-4}^{6} (2x - 2f(x))dx$ $= \int_{-4}^{6} 2x\, dx - 2\int_{-4}^{6} f(x)\, dx$

$= \left[x^2\right]_{-4}^{6} - 2 \times 10$ ✓

$= 36 - 16 - 20$

$= 20 - 20$ ✓

$= 0$

b) $\int_{-2}^{8} 3f(x-2)\, dx$

$= 3\int_{-2}^{8} f(x-2)\, dx$ translate $f(x)$ 2 units right

$= 3\int_{-4}^{6} f(x)\, dx$ ✓

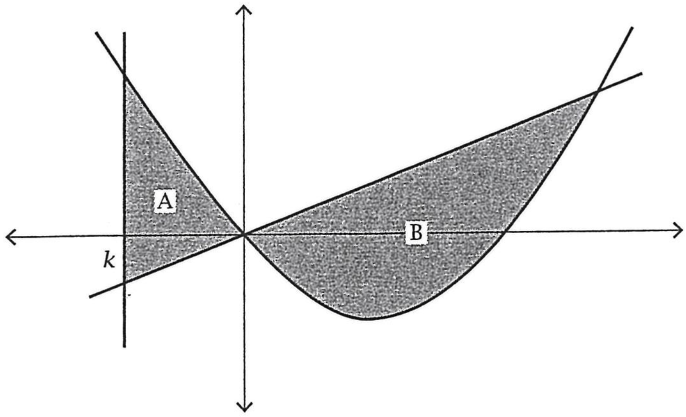
$= 3 \times 10$ ✓

$= 30$

Question 6 [3, 2, 3 = 8 marks]

The graph below consists of the following functions:

$$y = x^2 - 2x\,,\; y = \tfrac{1}{2}x\,,\; y = k \quad \text{where } k \text{ is a constant}$$



a) State an integral which represents the area of region B and calculate the area.

Solve $x^2 - 2x = \frac{1}{2}x$

$x = 0$, or $2\frac{1}{2}$ ✓

$A = \int_0^{2.5} \left(\frac{1}{2}x - (x^2 - 2x)\right) dx$ ✓

$= 2.604$ units^2 ✓

b) State an integral which represents the area of region A.

$A = \int_k^0 \left(x^2 - 2x - \frac{1}{2}x\right) dx$ ✓

$A = \int_k^0 \left(x^2 - \frac{5x}{2}\right) dx$

c) Find the value of k for which the area of region A equals the area of region B.

$\int_k^0 \left(x^2 - \frac{5x}{2}\right) dx = 2.604$ ✓

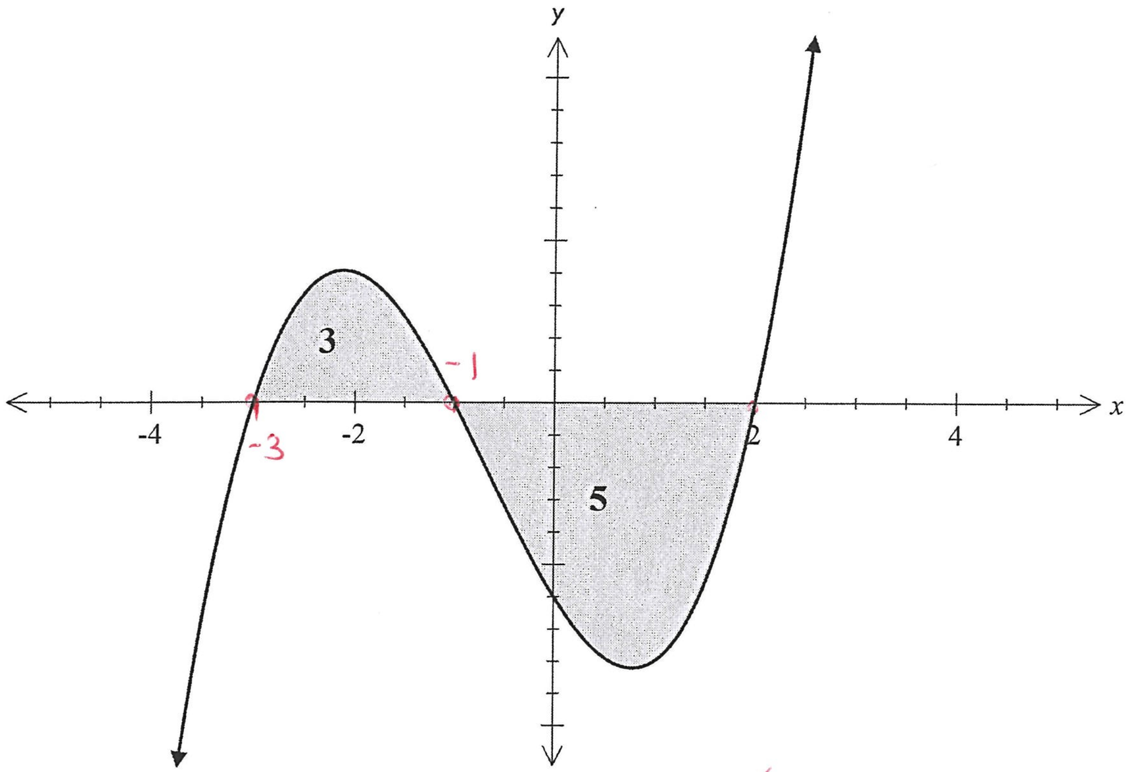
$\left[\frac{x^3}{3} - \frac{5x^2}{4}\right]_k^0 = 2.604$ ✓

Solve $-\left(\frac{k^3}{3} - \frac{5k^2}{4}\right) = 2.604$

$k = -1.025$ ✓

Question 7 [1, 1, 1, 1, = 4 marks]

Use the graph below to determine the following definite integrals. The area of the curve containing A is 3 square units and the area of the curve containing B is 5 square units.



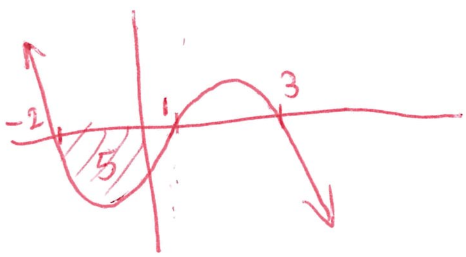
a) $\int_{-3}^{2} f(x)dx$ $= 3 + -5$

$= -2$

b) $\int_{1}^{-2} f(-x)dx$ $\Rightarrow$ reflection in y-axis

$= -\int_{-2}^{1} f(-x)dx = -(-5)$

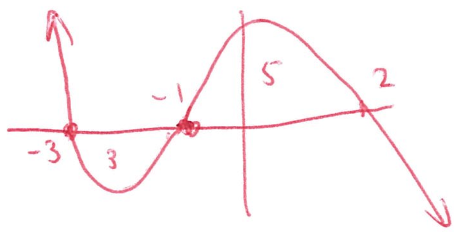
$= 5$



c) $\int_{-3}^{2} -f(x)dx$ $\Rightarrow$ reflection in x-axis

$= -3 + 5$

$= 2$



d) $\int_{-3}^{-1} [f(x) + 3]dx$ $= \int_{-3}^{-1} f(x) + \int_{-3}^{-1} 3\,dx$

$= 3 + [3x]_{-3}^{-1}$

$= 3 + (-3 - (-9))$

$= 3 + 6$

$= 9$

Question 8 [4 marks]

F T of C

Find $\int_0^4 \frac{d}{dx}\left(\frac{1-\sqrt{x}}{1+\sqrt{x}}\right)dx$ $= \left[\frac{1-\sqrt{x}}{1+\sqrt{x}}\right]_0^4$ ✓

$= \frac{1-2}{1+2} - \frac{1-0}{1+0}$ ✓

$= -\frac{1}{3} - 1$ ✓

$= -\frac{4}{3}$ ✓

OR $\int_0^4 \frac{-1}{\sqrt{x}(\sqrt{x}+1)^2}\,dx$ ✓

$= \left[\frac{2}{\sqrt{x}+1}\right]_0^4$ ✓

$= -\frac{4}{3}$ ✓✓

Question 9 [4 marks]

Find the x coordinate(s) of the stationary point(s) of the curve given by $y = \int_1^{x+1} t^2(t-2)\,dt$

$\frac{d}{dx}\left(\int_1^{x+1} t^2(t-2)\,dt\right)$

$= (x+1)^2(x+1-2)\times(1)$ ✓

$= (x+1)^2(x-1)$ ✓

stny pts when $\frac{dy}{dx} = 0$

Solve $(x+1)^2(x-1) = 0$

$x = \pm 1$ ✓✓

OR $\frac{d}{dx}\int_1^{x+1} (t^3 - 2t^2)\,dt$ ✓

$= \left((x+1)^3 - 2(x+1)^2\right).(1)$

$= (x+1)^2(x+1-2)$ ✓

$= (x+1)^2(x-1)$

$\frac{dy}{dx} = 0$

etc etc ✓✓

✓ OR $y = \frac{(x+1)^4}{4} - \frac{2(x+1)^3}{3} + \frac{5}{12}$

✓ $\therefore \frac{dy}{dx} = (x+1)^3 - 2(x+1)^2$

✓ $= (x+1)^2(x+1-2)$

$= (x+1)^2(x-2)$

✓ $\frac{dy}{dx} = 0$ etc etc.

OR

Solve ✓

✓ $0 = \frac{d}{dx}\left(\int_1^{x+1} t^2(t-2)\,dt\right)$

$x = -1$ or $x = 1$

✓ ✓

END OF TEST